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MECHANICS.

75. Proposed by B. F. FINKEL, A.M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A particle, P , is held in a bent tube by two forces directed towards two fixed points, H and S . Show that the equation of the tube is $PS \cdot PH = k^2$, if the forces are μ/PS and μ/PH .

III. Solution by G. B. M. ZERRE, A. M., Ph. D., Professor of Science and Mathematics, Chester High School, Chester, Pa.

Let $PS = r$, $PH = r'$, $\mu/PS = f$, $\mu/PH = f'$.

By the principle of virtual work we have for equilibrium,

$$fdr + f'dr' = 0, \text{ but } f/f' = r'/r \text{ or } f'r' = fr.$$

Dividing $fdr = -f'dr'$ by $fr = f'r'$ we get

$$dr/r = -dr'/r' \text{ or } rdr' + r'dr = 0.$$

Integrating, we get $rr' = \text{a constant} = k^2$.

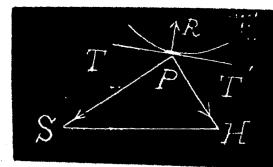
$$\therefore PS \cdot PH = k^2.$$

IV. Solution by GEORGE LILLEY, Ph.D., Professor of Mathematics, University of Oregon, Eugene, Oregon.

Let P be any position of the particle, TT' the tangent to the tube at P , $\angle TPS = \phi$, $\angle TPH = \phi'$, $\angle PSH = \theta$, $\angle PHS = \theta'$, $PS = r$ and $PH = r'$.

$$\text{Resolve along } TT', \frac{\mu}{r} \cos \phi + \frac{\mu}{r'} \cos \phi' = 0.$$

$$\text{But, } \tan \phi = r \frac{d\theta}{dr}; \text{ hence, } \cos \phi = \frac{dr}{ds}, \text{ where } ds$$



is element length of the tube. Also $\cos \phi' = \frac{dr'}{ds}$.

$$\text{Therefore, } \frac{dr}{2r} + \frac{dr'}{2r'} = 0.$$

Integrating, $\log \sqrt{(rr')} + c = 0$, where c depends on known values of r and r' . Therefore, $SP \cdot PH = k^2$.

Or thus : By the method of virtual work,

$$Fdr + F'dr' = 0, \text{ where } F = \frac{\mu}{SP} \text{ and } F' = \frac{\mu}{HP}.$$

Thus form the differential equation and solve it as above.

V. Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Miss.

Let P be the particle, S and H the fixed points. Denote PS , PH , and SH by h , s , and $2m$, respectively. Take the origin of rectangular axes midway between S and H , the x -axis lying along the line SH . Let the resultant force

acting at $P(x, y)$ intersect Ox at A , and make angles α, β, θ , with PS, PH , and AH , respectively.

Since $\frac{\mu}{SP} : \frac{\mu}{HP} = \sin\beta : \sin\alpha$, $HP : SP = \sin\beta : \sin\alpha$,

$$\text{or } \frac{\sin \alpha}{\sin \beta} = \frac{h}{s} \quad \dots \dots \dots \quad (1).$$

$$\text{Also } \frac{s \cdot \sin \beta}{h \cdot \sin \alpha} = \frac{AH}{AS} = \frac{m - OA}{m + OA}.$$

Substitute from (1) and solve for OA , obtaining

$$OA = \frac{h^2 - s^2}{h^2 + s^2} m. \quad \tan \theta = \frac{y}{x - OA} = \frac{y}{x - \frac{h^2 - s^2}{h^2 + s^2} m} = \frac{y^3 + x^2 y + m^2 y}{x^3 + x y^2 - m^2 x}.$$

Since PA is normal to the tube, the differential equation of the curve is

$$\frac{dy}{dx} = -\frac{x^3 + xy^2 - m^2 x}{y^3 + x^2 y + m^2 y}.$$

Integrating, $y^4 + x^4 + 2x^2y^2 + 2m^2y^2 - 2m^2x^2 = c$.

Adding m^4 to both members, and factoring,

$$[y^2 + (m+x)^2][y^2 + (m-x)^2] = c + m^4, \text{ or } SP^2 \cdot HP^2 = c + m^4 = k^4,$$

from which $SP \cdot HP = k^2$.

VI. Solution by R. E. GAINES, Professor of Mathematics, Richmond College, Richmond, Va.

Denote PS and PH by r and r' , respectively, and these may be taken as the "bipunctual coördinates" of P . Then it is easy to show that

$$\frac{dr'}{ds} = \cos\psi \text{ and } \frac{dr}{ds} = -\cos\varphi.$$

$$\therefore \frac{dr'}{dr} = - \frac{\cos\psi}{\cos\varphi}.$$

Now resolving forces along the tangent at P we have,

$$\frac{\mu}{r} \cos \varphi = \frac{\mu}{r'} \cos \psi \text{ or } \frac{1}{r} - \frac{1}{r'} \frac{\cos \psi}{\cos \varphi} = 0.$$

$$\therefore \frac{1}{r} + \frac{1}{r'} \frac{dr'}{dr} = 0.$$

$$\therefore \log r + \log r' = \log k^2, \quad \therefore rr' = k^2.$$

If the forces had been $\mu f(r)$ and $\mu F(r')$ we could get the form of the curve by integrating

$$f(r) + F(r') \frac{dr'}{dr} = 0.$$

76. Proposed by JAMES F. LAWRENCE, Classical Sophomore, Drury College, Springfield, Mo.

An inclined plane of mass M is capable of moving freely on a smooth horizontal plane. A perfectly rough sphere of mass m is placed on its inclined face and rolls down under the action of gravity. If x' be the horizontal space advanced by the incline plane, x the part of the plane rolled over by the sphere, prove that $(M+m)x' = mx\cos\alpha$, $\frac{1}{2}x - x'\cos\alpha = \frac{1}{2}gt^2\sin\alpha$, where α is the inclination of the plane. [From Routh's *Elementary Rigid Dynamics*, page 126.]

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio State University, Athens, Ohio.

Let F =friction of the sphere and plane, R =their mutual reaction, θ =the angle through which the sphere has rotated from the beginning of motion, y =the vertical distance of the center of the sphere from the horizontal plane, x_1 =the corresponding abscissa, h and k the initial values of x and y , respectively, and a =the radius of the sphere.

For the motion of the sphere, resolving horizontally and vertically, and taking moments about the center of the sphere,

$$m \frac{d^2x_1}{dt^2} = F\cos\alpha - R\sin\alpha \dots \dots \dots (1), \quad m \frac{d^2y}{dt^2} = F\sin\alpha + R\cos\alpha - mg \dots \dots \dots (2),$$

$$\frac{2}{3}ma^2 \frac{d^2\theta}{dt^2} = aF \dots \dots \dots (3).$$

For the horizontal motion of the plane,

$$M \frac{d^2x'}{dt^2} = -F\cos\alpha + R\sin\alpha \dots \dots \dots (4).$$

$$\text{Also, } x_1 = h + x' - a\theta\cos\alpha \dots \dots \dots (5), \quad y = k - a\theta\sin\alpha \dots \dots \dots (6).$$

$$\text{From (5) and (1), } m \frac{d^2x'}{dt^2} - mac\cos\alpha \frac{d^2\theta}{dt^2} = F\cos\alpha - R\sin\alpha \dots \dots \dots (7);$$

$$\text{and from (6) and (2), } -ma\sin\alpha \frac{d^2\theta}{dt^2} = F\sin\alpha + R\cos\alpha - mg \dots \dots \dots (8).$$

Eliminating F and R from (3), (7) and (8),

$$mac\cos\alpha \frac{d^2x'}{dt^2} = \frac{2}{3}ma \frac{d^2\theta}{dt^2} - mgs\sin\alpha \dots \dots \dots (9).$$

Integrating (9), noticing that when $t=0$, $\frac{dx'}{dt}=0$, $\frac{d\theta}{dt}=0$, and $x'=0$, $\theta=0$,